A Reciprocity Formulation for the EM Scattering by an Obstacle Within a Large Open Cavity

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Abstract— **A formulation based on a generalized reciprocity theorem is developed for analyzing the external high frequency EM scattering by a complex obstacle inside a relatively arbitrary open-ended waveguide cavity when it is illuminated by an external source. This formulation is also extended to include EM fields whose time dependence may be non-periodic. A significant advantage of this formulation is that it allows one to break up the analysis into two independent parts; one deals with the waveguide cavity shape alone and the other with the obstacle alone. Thus, it is useful for independently estimating the scattering effects due to modifications in the waveguide cavity shape for a given type of large complex obstacle, and due to different types of complex obstacles for a given type of large open waveguide cavity shape, respectively, without requiring one to treat the entire configuration each time one of these is changed. The external scattered field produced by the obstacle (in the presence of the waveguide cavity structure) is given in terms of a generalized reciprocity integral over a surface S_T corresponding to the interior waveguide cavity cross-section located conveniently but sufficiently close to the obstacle. Furthermore, the fields coupled into the cavity from the source in the exterior region generally need to propagate only one-way via the open front end (which is directly illuminated) to the interior surface S_T in this approach, and not back, in order to find the external field scattered by the obstacle.**

I. INTRODUCTION

A FORMULATION based on a generalized reciprocity theorem is developed for analyzing the high frequency electromagnetic (EM) scattering by relatively arbitrary open-ended waveguide cavities containing a large complex interior obstacle or termination. An extension of this formulation to include EM fields with non-periodic or arbitrary time dependence is also presented. These results are of significant interest in scattered field and EM coupling predictions. An important advantage of the formulation developed here is that it allows one to independently estimate the effects on the overall cavity-obstacle scattering due to modifications in the waveguide cavity shape for a given interior obstacle, and due to different obstacles for a given open waveguide cavity shape, respectively, without having to analyze the entire cavity-obstacle configuration each time one of them (i.e., the cavity shape or the obstacle) is changed. The latter aspect will be discussed in more detail in a separate paper.

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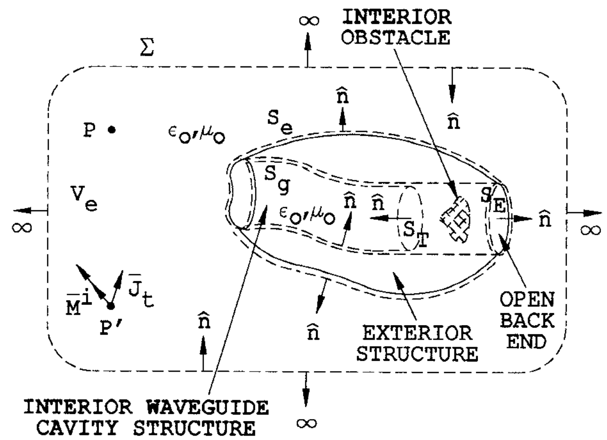

Fig. 1. Original problem configuration.

A typical geometry of the general problem under consideration is depicted in Fig. 1. The geometry is illuminated by an external current source (at P'), and the observer is also assumed to be in the external region (at P). It is primarily of interest in this study to be able to analyze the external scattering from a geometry of the type in Fig. 1 for cases where the open front end of the cavity is directly illuminated by the source, and for observation points which are also in direct view of the open front end, as shown in the figure. Furthermore, the medium surrounding the cavity structure is taken to be free space and the external surface as well as the interior cavity walls are assumed to be impenetrable (e.g. perfectly conducting walls with or without material coating). S_T is an arbitrary surface which either encloses the interior obstacle or partitions the obstacle/termination region from the rest of the open-ended waveguide region (as in Fig. 1), and S_E is the surface defined by the open back end of the cavity beyond the obstacle. It is noted that as a special case, the back end of the cavity (at S_E) could be closed, or the obstacle itself could form a termination which completely closes the back end of the cavity. Furthermore, the waveguide region beyond the obstacle could also, as a special case, be made semi-infinite. These latter special cases of the more general situation depicted in Fig. 1 are discussed in Section II.

The formulation for the field scattered into the exterior region by just the interior obstacle is based on a generalized reciprocity integral which requires a knowledge of the fields on the surfaces S_T and S_E due to the illumination from the original current source (at P') with the obstacle present, and it also requires a knowledge of the fields on S_T and S_E due to a conveniently chosen impressed test current source placed at the observer location (at P) but in the absence of the obstacle and with the original source turned off. This reciprocity integral which exists over S_T and S_E is shown in the next section to furnish the field scattered into the exterior by the obstacle in the presence of the waveguide cavity structure. Such a formulation has the additional advantage that, in most cases of practical interest, the fields coupled into the cavity from the sources in the exterior region need to propagate only one-way (in the forward direction) via the open front end to the interior surface S_T, and not back (in the reverse direction), in order to find the external field scattered by the obstacle. More will be said about this property later.

The development of the generalized reciprocity integral is given in Section II, and Section III discusses some methods for finding the relevant field quantities which appear within this integral. Section IV presents some numerical results based on this development, and compares them with the corresponding solutions obtained without the use of the generalized reciprocity integral for the sake of establishing an independent check. An $e^{j\omega t}$ time convention for the fields and sources is assumed and suppressed for the periodic or time harmonic case. Also, the cavity-obstacle configuration is assumed to be embedded in free space.

II. Generalized Reciprocity Integral for Time Harmonic Interior Obstacle Scattered Fields

Consider the open-ended waveguide cavity configuration illustrated in Fig. 1 which is illuminated by an external impressed electric current source $\overline{J}^i(P')$ and a magnetic current source $\overline{M}^i(P')$ at P'. Let $(\overline{E}_c^i, \overline{H}_c^i)$ denote the (electric, magnetic) fields which are produced by these sinusoidally time varying impressed sources $\overline{J}^i(P')$ and $\overline{M}^i(P')$ when the cavity structure is present but with the interior obstacle absent. The $\overline{J}^i(P')$ and $\overline{M}^i(P')$ radiating in the presence of the cavity structure and the obstacle produce the fields $(\overline{E}, \overline{H})$ where

$$\overline{E} = \overline{E}_c^i + \overline{E}_c^s \tag{1}$$

$$\overline{H} = \overline{H}_c^i + \overline{H}_c^s \tag{2}$$

and $(\overline{E}_c^s, \overline{H}_c^s)$ therefore denote the fields scattered by just the interior obstacle but in the presence of the cavity walls. Note that the above fields satisfy the following Maxwell's Curl equations:

$$\left\{\begin{array}{l} \nabla \times \overline{E} = -j\omega\mu_o \overline{H} - \overline{M}^i(P') \\ \nabla \times \overline{H} = \overline{J}^i(P') + j\omega\epsilon_o \overline{E} \end{array}\right\} \tag{3}$$

$$\left\{\begin{array}{l} \nabla \times \overline{E}_c^i = -j\omega\mu_o \overline{H}_c^i - \overline{M}^i(P') \\ \nabla \times \overline{H}_c^i = \overline{J}^i(P') + j\omega\epsilon_o \overline{E}_c^i \end{array}\right\}, \tag{4}$$

and hence,

$$\left\{\begin{array}{l} \nabla \times \overline{E}_c^s = -j\omega\mu_o \overline{H}_c^s \\ \nabla \times \overline{H}_c^s = j\omega\epsilon_o \overline{E}_c^s \end{array}\right\}. \tag{5}$$

It is of primary interest to find $(\overline{E}_c^s, \overline{H}_c^s)$ at any external point P when P is on the same side of the cavity as the original source at P'. The fields $(\overline{E}_c^s, \overline{H}_c^s)$ can be found in terms of a set of equivalent sources on S_T and S_E (of Fig. 1) along with

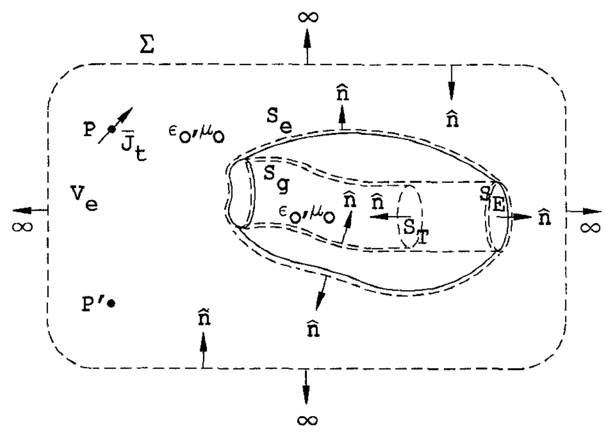

Fig. 2. Related test problem configuration.

a set of test fields, $(\overline{E}_t, \overline{H}_t)$, which are produced by an electric current test source $\overline{J}_t(P)$ at P that has the same frequency as $(\overline{J}^i, \overline{M}^i)$ when it radiates with the cavity structure present but with the interior obstacle absent, as illustrated in Fig. 2. The fields $(\overline{E}_t, \overline{H}_t)$ satisfy the following Maxwell's Curl equations:

$$\left\{\begin{array}{l} \nabla \times \overline{E}_t = -j\omega\mu_o \overline{H}_t \\ \nabla \times \overline{H}_t = \overline{J}_t(P) + j\omega\epsilon_o \overline{E}_t \end{array}\right\} \tag{6}$$

The fields $(\overline{E}_c^s, \overline{H}_c^s)$ can be related to the fields $(\overline{E}_t, \overline{H}_t)$ via the divergence theorem applied to the quantity $\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s$ within the volume V_e which is bounded by the surfaces $[S_T + S_E + S_e + S_g + \Sigma]$ as shown in Figs. 1 and 2. Thus,

$$\int_{V_e} \nabla \cdot (\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s)\, dv = -\oint_{\Sigma + S_e + S_g + S_T + S_E} (\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s) \cdot \hat{n}\, ds \tag{7}$$

where $\hat{n}$ is the unit normal vector which points into the region V_e. Using (5) and (6), the L.H.S. of (7) reduces, via the radiation and boundary conditions together with some vector algebra, to

$$\int_{V_e} \overline{J}_t(P) \cdot \overline{E}_c^s\, dv'' = \int_{S_T + S_E} (\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s) \cdot \hat{n}\, ds. \tag{8}$$

It is noted that the fields satisfy the radiation condition on Σ as $\Sigma \to \infty$, hence the integral on Σ vanishes in (7). For perfectly conducting walls, both $\hat{n} \times \overline{E}_c^s$ as well as $\hat{n} \times \overline{E}_t$ vanish on $S_e + S_g$, so that the integrals on those boundaries also vanish in (7); on the other hand if these walls are coated with absorbing layers, then $S_e + S_g$ is taken to be on the conducting walls, whereas, if the walls are impenetrable then S_e and S_g can be made to lie just within the impenetrable wall of some thickness (however small), so that the integrals on S_e and S_g can be made to vanish again in (7). This leaves one with integrals only over S_T and S_E on the R.H.S. of (7), thereby leading directly to the expression on the R.H.S.

of (8). The result in (8) constitutes a generalized reciprocity relationship because $(\overline{E}_c^s, \overline{H}_c^s)$ and $(\overline{E}_t, \overline{H}_t)$ are evaluated in different environments, i.e., $(\overline{E}_c^s, \overline{H}_c^s)$ are found with the interior obstacle present while $(\overline{E}_t, \overline{H}_t)$ are found with the interior obstacle absent. In contrast, the standard reciprocity theorem [1] relates a pair of fields (due to a pair of sources) in the same environment.

Let the test current $\overline{J}_t(P)$ be a point source of strength $\overline{P}_t$; thus,

$$\overline{J}_t(P) = \overline{P}_t \delta(\overline{r}'' - \overline{r}) \tag{9}$$

where $\overline{r}$ is the position vector of the observation point at P and $\overline{r}''$ is the variable of integration (in V_e) on the L.H.S. of (8). Now, from (8) and (9) one obtains,

$$\overline{P}_t \cdot \overline{E}_c^s(\overline{r}) = \int_{S_T + S_E} (\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s) \cdot \hat{n}\, ds. \tag{10}$$

When the source and observer are in direct view of the open front end, as in the case shown in Fig. 1, then the contribution to $\overline{E}_c^s(\overline{r})$ at P from the integration over S_E in (10) is, in general, sufficiently small in comparison to that from the integration over S_T for a relatively large obstacle, as is assumed to be the case here. Therefore, (10) can be approximated in this case by:

$$\overline{P}_t \cdot \overline{E}_c^s(\overline{r}) \cong \int_{S_T} (\overline{E}_c^s \times \overline{H}_t - \overline{E}_t \times \overline{H}_c^s) \cdot \hat{n}\, ds. \tag{11}$$

It is noted that (11) is obtained exactly if the cavity is closed at the end S_E, or if the obstacle is assumed to totally block S_E from S_T. One can also arrive at (11) exactly if the surface S_E is allowed to recede to infinity so that the open-ended cavity configuration in Fig. 1 becomes semi-infinite (as S_E recedes to ∞). In the latter case, one must impose a physical requirement that there are only outgoing waves crossing S_E and no waves incoming (or reflected back) into the cavity from S_E as $S_E \to \infty$. This in turn implies that the waveguide cavity region near and at S_T must be assumed to be uniform (i.e., with a constant cross section) if $S_E \to \infty$; one can then define an orthogonal set of waveguide modes at S_E and express $(\overline{E}_c^s, \overline{H}_c^s)$ as well as $(\overline{E}_t, \overline{H}_t)$ in terms of these modes within the uniform waveguide region. It follows from modal orthogonality that the integral over S_E (as $S_E \to \infty$) vanishes in (10) for the latter case thereby leading to the desired result in (11). On the other hand, if the waveguide cavity is made lossy (or even slightly lossy) as $S_E \to \infty$, then the integral over S_E in (10) vanishes once more thereby leading again to (11). Furthermore, if it is assumed that the interior reflection of the waves back into the cavity from the electrically large open front end is small, then $(\overline{E}_e^s, \overline{H}_e^s)$ at S_T may be approximated simply by the fields denoted by $(\overline{E}_o^s, \overline{H}_o^s)$ within the cavity which are scattered by the obstacle, but which exclude the effects of all multiple wave interactions between the obstacle and the open front end. Likewise, one may approximate $(\overline{E}_t, \overline{H}_t)$ at S_T in (11) by the fields denoted as $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ which arrive directly at S_T from $\overline{J}_t(P)$ via the open front end, but which exclude any contributions arriving from $\overline{J}_t(P)$ via the open end at S_E in Fig. 2 and which also exclude any effects of multiple wave interactions between the open front end and the back end S_E; therefore, $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ are found by tracking the fields one way from $\overline{J}_t(P)$ at P to S_T via the open front end. Finally, under the above approximations which are assumed to hold true, (11) leads to the following desired result for the field $\overline{E}_c^s(P)$ scattered at P by the interior obstacle when the cavity-obstacle configuration of Fig. 1 is illuminated externally by $\overline{J}^i(P')$; namely,

$$\overline{E}_c^s(P) \cdot \overline{P}_t \approx \int_{S_T} (\overline{E}_o^s \times \overline{H}_t^{ig} - \overline{E}_t^{ig} \times \overline{H}_o^s) \cdot \hat{n}\, dS. \tag{12}$$

It is noted that the $\overline{E}_c^s(P)$ on the L.H.S. of (12) can be found via the R.H.S. of (12) in terms of $(\overline{E}_o^s, \overline{H}_o^s)$ and $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$, both of which need to be evaluated only over the interior surface S_T near the obstacle. An alternative form of (12) can be expressed as:

$$\overline{E}_c^s(P) \cdot \overline{P}_t \approx \oint_{S_o} (\overline{E}_o^s \times \overline{H}_t^{ig} - \overline{E}_t^{ig} \times \overline{H}_o^s) \cdot \hat{n}\, dS \tag{13}$$

where the integration is over a closed surface S_o which encapsulates the obstacle.

III. On the Evaluation of $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ and $(\overline{E}_o^s, \overline{H}_o^s)$ at S_T for the Time- Harmonic Case

For relatively arbitrary cavities and for high frequencies, $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ in (12) can be evaluated, for example, by the shooting and bouncing ray (SBR) technique [2]–[5], the Gaussian beam (GB) shooting method [4], [5] or the generalized ray expansion (GRE) technique [5], [6]. As mentioned in the introduction, the use of (12) requires that the fields from the exterior sources at P and P' need to propagate only one-way via the open front end to S_T and not back. Furthermore, the GB/GRE methods require shooting a set of beams/rays only once from the open front end since the launching directions of these beams/rays and hence the propagation paths within the cavity are independent of the source location (i.e., whether the excitation be at the original source at P' or be at the observation point P for generating $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$); only the initial beam/ray amplitudes depend on the excitation. The fields $(\overline{E}_o^s, \overline{H}_o^s)$ can be found by first obtaining $(\overline{E}^{ig}, \overline{H}^{ig})$ at S_T, which are the fields incident from the original source $\overline{J}_i(P')$ at P' in the absence of the interior cavity obstacle; $(\overline{E}^{ig}, \overline{H}^{ig})$ are found in exactly the same manner as $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ and are thus based on the same assumptions and approximations as those required to find $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$. It may be possible that the interior reflection from some types of obstacles can be analyzed using ray methods, in which case the ray fields $(\overline{E}^{ig}, \overline{H}^{ig})$ enter the cavity after being excited by the original source $\overline{J}_i(P')$ and continue beyond S_T into the obstacle-cavity region to subsequently reflect back from the obstacle to S_T as $(\overline{E}_o^s, \overline{H}_o^s)$. In the event that an analytical approach based on ray methods either cannot be used or does not easily lend itself to find $(\overline{E}_o^s, \overline{H}_o^s)$, it may be possible to employ a numerical approach to accomplish this task. Such a numerical approach may be based on a partial differential equation solution of the wave problem using the finite element or finite difference methods, or the integral equation solution based on the method of moments, or a hybrid combination of both methods to provide $(\overline{E}_o^s, \overline{H}_o^s)$ once $(\overline{E}^{ig}, \overline{H}^{ig})$ is given. In these numerical methods, it would be worth employing the "Green's function" for the cavity without the obstacle in the region beyond S_T which would otherwise contain the obstacle, so that only the fields (or currents) induced in/on the obstacle would need be found, because the presence of the cavity walls is automatically accounted for by this Green's function. Furthermore, the Green's function for the cavity without the obstacle can be represented locally by an eigenfunction expansion for waveguide cavities with, for example, a circular cross section in the region where the obstacle would otherwise be present, or be approximated via ray methods in the case of arbitrary cavities for which modes cannot be defined in the usual manner. If neither the analytical nor the numerical methods can be employed effectively to find $(\overline{E}_o^s, \overline{H}_o^s)$, as may be the case for highly complex and electrically large obstacles, then alternative (e.g. experimental) methods must be employed.

It is noted that the $(\overline{E}_o^s, \overline{H}_o^s)$ can also be found, in principle, via a different approach which employs any of the aforementioned techniques such as the ray methods, numerical methods or other alternative (e.g. experimental) techniques to develop a local Green's function for the obstacle-cavity region contained between S_T and S_E with the obstacle present. This local obstacle-cavity Green's function would provide the response at S_T, due to a point source also located in the same plane S_T and with the obstacle present. Such a Green's function can be constructed approximately, but with sufficient accuracy, to emphasize only the local cavity-obstacle region between S_T and S_E; it would then also furnish the obstacle response $(\overline{E}_o^s, \overline{H}_o^s)$ at S_T due to an excitation $(\overline{E}^{ig}, \overline{H}^{ig})$ at S_T due to the original source $\overline{J}_i$ at P'. The evaluation of $(\overline{E}^{ig}, \overline{H}^{ig})$ and $(\overline{E}_t^{ig}, \overline{H}_t^{ig})$ on S_T are totally dependent on the long waveguide cavity shape from the open front end (directly illuminated by $\overline{J}_i(P')$ and $\overline{J}_t(P)$, respectively) to the fictitious plane S_T; whereas, the local cavity-obstacle Green's function alluded to above (and which plays a role in furnishing $(\overline{E}_o^s, \overline{H}_o^s)$) depends primarily on the short cavity section between S_T and S_E containing the obstacle. Thus, one can separate the effects of the short obstacle region of the cavity from the rest of the cavity, and indeed very effectively ascertain how a given obstacle affects a variety of long waveguide cavity shapes connected to the short part of the cavity containing the obstacle, and vice versa. Yet another different, but related, approach which separates the analysis of the shape dependent cavity region from the obstacle region is described in [7]–[9].

IV. Generalized Reciprocity Integral for Interior Obstacle Scattered Fields for Arbitrary Time Dependent Excitation

The general result obtained in (8) of Section II for sinusoidally time varying (or time harmonic) fields can be extended directly to fields whose time dependence is arbitrary, as will be shown below. Indeed, a procedure for extending the frequency domain (or time harmonic) form of a reciprocity theorem as originally developed by Lorentz into a form valid for fields with non-periodic time dependence has been presented by Goubau [10]. The present procedure for the development of the time dependent form of (8) follows essentially from [10]. Since (8) represents a result which is valid for all frequencies (ω), it can be converted as usual into the time domain via the inverse Fourier transform defined by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\, d\omega \tag{14a}$$

where $f(t)$ is an arbitrary time dependent function synthesized from the frequency domain spectrum function $F(\omega)$. The $F(\omega)$ can be found from the direct Fourier transform of $f(t)$

$$F(\omega) = \int_{-\infty}^{\infty} f(\mathcal{T}) e^{-j\omega \mathcal{T}}\, d\mathcal{T}. \tag{14b}$$

The relationship in (14a) and (14b), between the transform pair $f(t)$ and $F(\omega)$, is commonly denoted by:

$$f(t) \leftrightarrow F(\omega). \tag{14c}$$

Next, employing the spectral inversion of (14a) to (8) yields

$$\frac{1}{2\pi} \int_{V_e} dv'' \int_{-\infty}^{\infty} d\omega\, e^{j\omega t} \overline{E}_c^s(\overline{r}''; \omega) \cdot \overline{J}_t(\overline{r}, \overline{r}''; \omega)$$
$$= \frac{1}{2\pi} \iint_{S_T+S_E} ds \int_{-\infty}^{\infty} d\omega\, e^{j\omega t} \hat{n} \cdot \Big[\overline{E}_c^s(\overline{r}; \omega) \times \overline{H}_t(\overline{r}; \omega) + \overline{H}_c^s(\overline{r}; \omega) \times \overline{E}_t(\overline{r}; \omega)\Big]_{\substack{\overline{r}\text{ on}\\ S_T+S_E}} \tag{15}$$

The orders of integration have been interchanged in (15). Following the notation in (14c), one may introduce the necessary time domain field quantities via the relations:

$$\overline{e}_c^s(\overline{r}; t) \leftrightarrow \overline{E}_c^s(\overline{r}; \omega); \tag{16a}$$
$$\overline{h}_c^s(\overline{r}; t) \leftrightarrow \overline{H}_c^s(\overline{r}; \omega); \tag{16b}$$
$$\overline{e}_t(\overline{r}; t) \leftrightarrow \overline{E}_t(\overline{r}; \omega); \tag{17a}$$
$$\overline{h}_t(\overline{r}; t) \leftrightarrow \overline{H}_t(\overline{r}; \omega). \tag{17b}$$
$$\overline{j}_t(\overline{r}, \overline{r}''; t) \leftrightarrow \overline{J}_t(\overline{r}, \overline{r}''; \omega). \tag{18}$$

At this juncture, it is useful to represent the $(\overline{E}_c^s, \overline{H}_c^s)$ spectral (frequency domain) values in (15) by the arbitrary time domain functions $(\overline{e}_c^s, \overline{h}_c^s)$ which they synthesize;

$$\int_{V_e} dv'' \int_{-\infty}^{\infty} d\omega\, e^{j\omega t} \int_{-\infty}^{\infty} d\tau\, e^{-j\omega\tau} \overline{e}_c^s(\overline{r''}\tau) \cdot \overline{J}_t(\overline{r}, \overline{r}''; \omega)$$
$$= \iint_{S_T+S_E} ds \int_{-\infty}^{\infty} d\omega\, e^{j\omega t} \int_{-\infty}^{\infty} d\tau\, e^{-j\omega\tau} \hat{n} \cdot \Big[\overline{e}_c^s(\overline{r}; \tau) \times \overline{H}_t(\overline{r}; \omega) + \overline{h}_c^s(\overline{r}; \tau) \times \overline{E}_t(\overline{r}; \omega)\Big]_{\substack{\overline{r}\text{ on}\\ S_T+S_E}} \tag{19a}$$

Performing the integration on ω in (19a) yields

$$\int_{V_e} dv \int_{-\infty}^{\infty} d\tau\, \overline{e}_c^s(\overline{r};\tau)\cdot \overline{j}_t(\overline{r},\overline{r};t-\tau)$$

$$= \iint_{S_T+S_E} ds \int_{-\infty}^{\infty} d\tau\, \hat{n}\cdot \Big[\overline{e}_c^s(\overline{r};\tau)\times \overline{h}_t(\overline{r};t-\tau) + \overline{h}_c^s(\overline{r};\tau)\times \overline{e}_t(\overline{r};t-\tau)\Big]_{\overline{r}\text{ on } S_T+S_E} \tag{19b}$$

If one assumes an impulsive behavior for $\overline{j}_t$ in both space and time, then:

$$\overline{j}_t(\overline{r},\overline{r}'';t) = \overline{P}_t\delta(\overline{r}-\overline{r}'')\,\delta(t). \tag{20}$$

It follows from (20) that $\overline{j}_t(\overline{r},\overline{r}'';t-\tau) = \overline{P}_t\delta(\overline{r}-\overline{r})\,\delta(t-\tau)$; incorporating this information into (19b) yields

$$\overline{e}_c^s(\overline{r};t)\cdot\overline{P}_t = \iint_{S_T+S_E} ds \int_{-\infty}^{\infty} d\tau\, \hat{n} \cdot \Big[\overline{e}_c^s(\overline{r};\tau)\times \overline{h}_t(\overline{r};t-\tau) + \overline{h}_c^s(\overline{r};\tau)\times \overline{e}_t(\overline{r};t-\tau)\Big]_{\overline{r}\text{ on } S_T+S_E} \tag{21}$$

The above result in (21), which is in the time domain, is the counterpart of (8) for the frequency domain. The L.H.S. of (21) can be found via a time convolution of the fields of the original arbitrarily time varying source located at P', in the presence of the cavity and obstacle, with the fields of a time impulsive point test source at P, in the presence of the cavity but in the absence of the obstacle. It is noted that the time convolutions are performed at each point in $S_T + S_E$; these are then superposed as evident from the integral over S_T+S_E on the R.H.S. of (21).

If one makes the approximations leading from (8) to (12), then one can likewise obtain a time-dependent form of (12) using the same procedure as above; thus:

$$\overline{e}_c^s(\overline{r};t)\cdot\overline{P}_t \approx \iint_{S_T} ds \int_{-\infty}^{\infty} d\tau\, \hat{n}\cdot \Big[\overline{e}_o^s(\overline{r};\tau)\times \overline{h}_t^{ig}(\overline{r};t-\tau) + \overline{h}_o^s(\overline{r};\tau)\times \overline{e}_t^{ig}(\overline{r};t-\tau)\Big]_{\overline{r}\text{ on } S_T} \tag{22a}$$

where

$$\overline{e}_o^s(\overline{r};t)\leftrightarrow \overline{E}_o^s(\overline{r};\omega); \qquad \overline{h}_o^s(\overline{r};t)\leftrightarrow \overline{H}_o^s(\overline{r};\omega) \tag{22b}$$

$$\overline{e}_t^{ig}(\overline{r};t)\leftrightarrow \overline{E}_t^{ig}(\overline{r};\omega); \qquad \overline{h}_t^{ig}(\overline{r};t)\leftrightarrow \overline{H}_t^{ig}(\overline{r};\omega) \tag{22c}$$

Since the result in (12) is obtained from (8) after using high frequency approximations, it is thus reasonable to expect that the time domain result for $\overline{e}_c^s(\overline{r},t)$ in (22a) (obtained from (12)) will provide a useful approximation to the time domain result for $\overline{e}_c^s(\overline{r},t)$ in (21) (obtained from (8)) only during the early to intermediate times of arrival of the signal $\overline{e}_c^s(\overline{r},t)$ which is observed at the point P. The quantities on the right side of (22a) may be found by transforming the corresponding frequency domain fields (see (22b) and (22c)) into the time domain; alternatively, they could be found directly in the time domain. The latter aspect will be discussed in more detail in a separate paper.

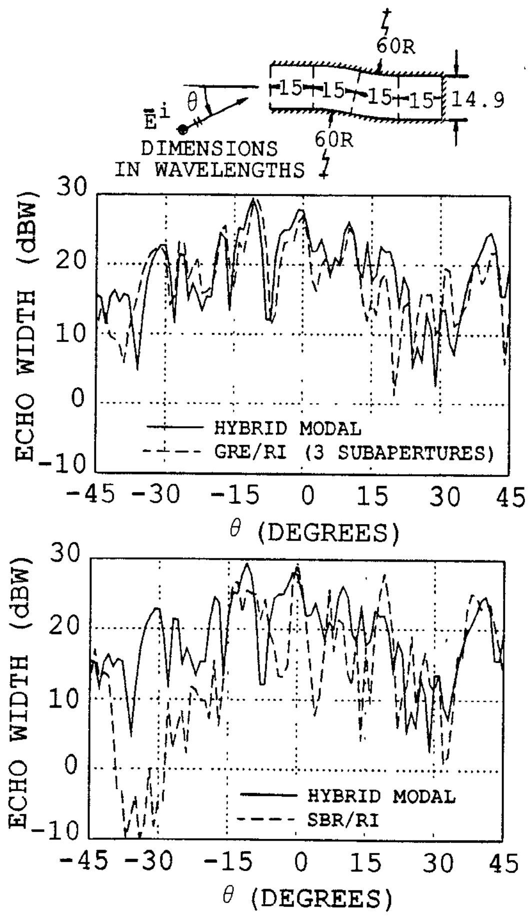

Fig. 3. Echo width versus aspect angle for a 2-D S-shaped open-ended cavity with a planar termination found using the reciprocity integral (RI) with the GRE and SBR methods.

V. Numerical Results

Figs. 3 and 4 show the EM echo width vs. aspect angle patterns of a perfectly conducting 2-D S-shaped open-ended waveguide cavity with a planar interior termination. The 2-D echo width σ is defined by

$$\sigma = \lim_{\rho\to\infty} 2\pi\rho \frac{|\overline{E}_c^s(\overline{\rho})|^2}{|\overline{E}^i|^2}, \tag{23}$$

where $\overline{\rho}$ is the vector to the far field observer (at P), $\overline{E}_c^s(\overline{\rho})$ is the field at $\overline{\rho}$ scattered by the interior termination of the cavity, as given by (12), and $|\overline{E}^i|$ is the magnitude of the plane wave field incident on the open front end (P' is located at infinity to create an incident plane wave). In Figs. 3 and 4, the echo width is given in decibels relative to a wavelength (DBW) (i.e., as 10 log σ with σ in free space wavelengths), and the incident electric field is polarized perpendicular to the plane of the geometry. It is noted that only the first order scattering from the interior of the cavity is shown in these figures. No external scattering or multiple wave interaction effects are included.

The solid line in the plots of Figs. 3 and 4 is calculated using the hybrid asymptotic-modal method [4], [5] and is used as a reference solution. The dashed lines are solutions based on the SBR [2]–[5] and GRE [5], [6] methods; in Fig. 3, the one-way tracking procedure of the generalized reciprocity integral of

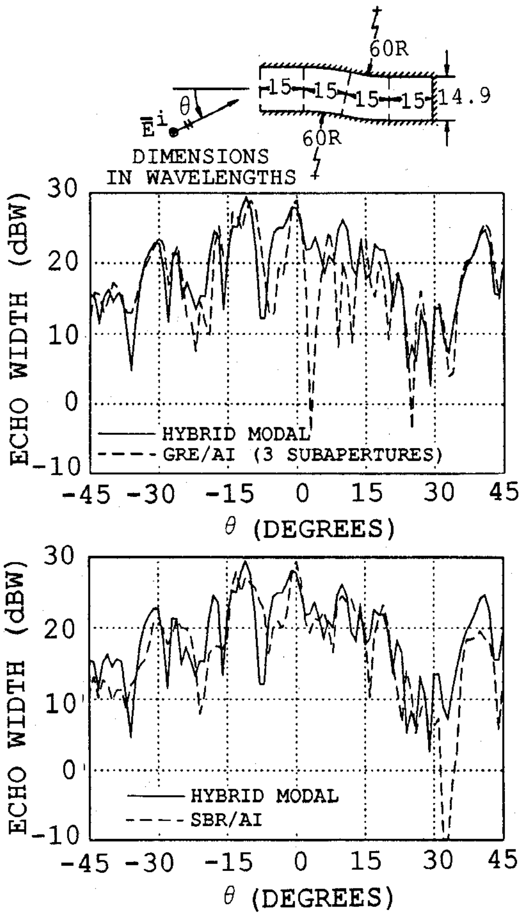

Fig. 4. Echo width versus aspect angle for a 2-D S-shaped open-ended cavity with a planar termination found using aperture integration (AI) with the GRE and SBR methods.

(12) is used, and in Fig. 4, the two-way tracking procedure of the aperture integration method is used. The numerical results in Fig. 3 which are based on the one way ray tracking that makes use of the reciprocity integral can be obtained almost twice as fast as the ones in Fig. 4 that require a two-way tracking. To compute the generalized reciprocity results of Fig. 3, the ray fields at the termination plane are converted into parallel plate waveguide modes and the orthogonality property of the modes is used to easily evaluate (12). Generally, the type of results in Fig. 3 can be obtained in less than a couple of minutes on, for example, a VAX 8550 computer.

This method can also be employed with the same degree of success for 3-D problems which are currently under study; the solutions to these will be reported later along with results for fields with non-periodic or arbitrary time dependence.

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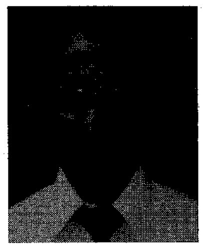

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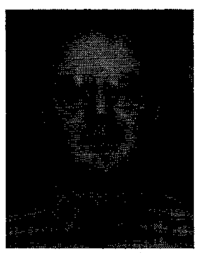

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